Motion Control	Console Position: TOPO
Velocity, Acceleration and Vectors	

State Vectors

How Does the International Space Station Stay in Orbit?

Instructional Objectives

Students will

- calculate the position and velocity vectors with respect to **x** and **y**, and
- determine the velocity and acceleration in the x and y direction for a set time interval.

Degree of Difficulty

This problem requires students to integrate several aspects of the AP Physics curriculum to obtain the solution. For the average AP Physics student, the problem may be moderately difficult.

Total Time Required

Teacher Prep Time: 5–10 minutes

Class Time: 60–80 minutes

(To decrease amount of class time, students may complete research as homework via the internet using the ISSLive! website or mobile application.)

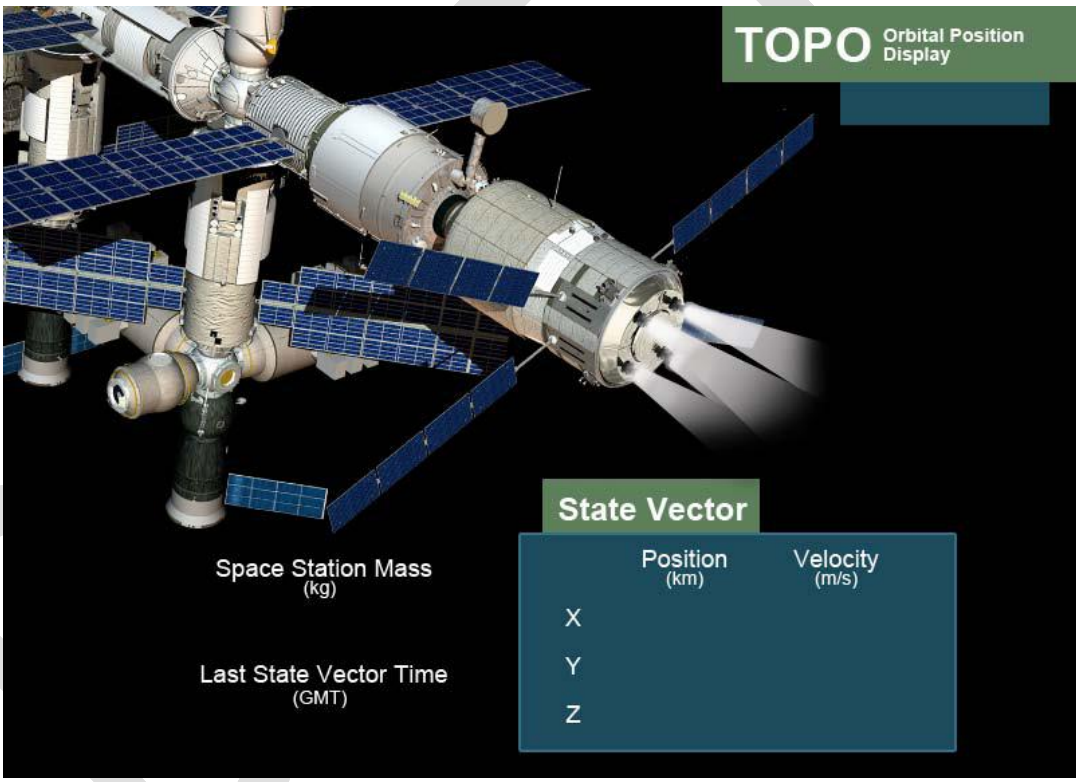
- Introduction: 5–10 minutes
- Student Research: 20–25 minutes
- Student Work Time: 25–30 minutes
- Post Conclusion: 10–15 minutes

Lesson Development

This problem is part of a series of problems associated with the NASA International Space Station *Live!* (ISS*Live!*) website at http://spacestationlive.jsc.nasa.gov.

Teacher Preparation

- Review the Motion Control information on the ISS*Live!* website. This may be found at the *Operations* tab, under *Core Systems*.
- Review the Trajectory Operations Officer (TOPO) Handbook, paying specific attention to the state vectors. This handbook may be found at the TOPO console position in the 3D Mission Control Center environment (under the *Interact* tab, then *Explore Mission Control*).
- Review the TOPO console display in the 3D Mission Control Center environment and the live data associated with state vectors. The displays may be accessed by clicking on the console screens.



TOPO Console Display

- Review the interactive activity at the TOPO console position in the 3D Mission Control Center environment *by clicking on the globe* on top of the console. This activity demonstrates how the state vectors can be used to determine ISS position and velocity.
- Prepare copies of the STUDENT WORKSHEET (Appendix B).

Inquiry-Based Lesson (Suggested Approach)

1. Pose this question to the class:

   How can position, velocity and acceleration be determined for the International Space Station (ISS)?

2. Allow students to discuss the question in small groups or as a class. Have students build their own questions and possible solutions to the problem.
3. Distribute the STUDENT WORKSHEET to the class. Students may work individually or in small groups (2–3 members per group) to conduct the research. This may be assigned as homework.
4. In order to conduct the research, students should access the ISS*Live!* website and explore the 3D Mission Control Center. If needed, guide students to the TOPO console position. They should access the TOPO Handbook and TOPO console displays, as well as the interactive activity, as they prepare to answer the questions on the STUDENT WORKSHEET.
5. Once the research is completed, students may work individually to complete the questions on the STUDENT WORKSHEET. They should refer to the live data on the TOPO console displays located on the ISS*Live!* website to answer the entire problem.

Post Conclusion

6. A SOLUTION KEY (Appendix A) is provided below using data that is typical for state vectors under normal conditions. Students' answers will vary depending on the actual live data.
7. Have students discuss their answers in small groups or with the entire class and tie back to the original question:

   How can position, velocity and acceleration be determined for the International Space Station (ISS)?

8. Ask students to explain the Motion Control System and the data they used in their calculations.
9. Assessment of student work may be conducted by using the provided rubric (modeled after AP Free Response Question scoring).

Extension

Other possible uses for the ISS*Live!* website, focusing on TOPO and the Motion Control System:

- Revisit the TOPO console position to check the live data at different time intervals or the ISS location, and compare the position, velocity and acceleration.
- Revisit the TOPO console position and check the live data during activities to compare how the position, acceleration and velocity may be adjusted to accommodate events such as visiting vehicles.

AP Course Topics

Circular Motion and Rotation

- Angular velocity
- Angular acceleration
- Linear acceleration
- Force between two objects
- Comparison between angular component and linear components
- Component vectors

NSES Science Standards

Inquiry

- Abilities necessary to do scientific inquiry
- Understanding about scientific inquiry

Science in Personal and Social Perspectives

- Science and technology in local, national and global challenges

Physical Science

- Circular motion
- Gravity

Science and Technology

- Abilities of technological design
- Understanding about science and technology

History and Nature of Science

- Science as a human endeavor
- Nature of scientific knowledge

Contributors

This problem is part of a series of problems developed by the ISS*Live!* Team with the help of NASA subject matter experts.

Education Specialist
Todd Morstein – Glacier High School, Kalispell School District, MT

NASA Expert
Ryan East – Trajectory Analyst, International Space Station Mission Control Center, NASA Johnson Space Center

Scoring Guide

Suggested 15 points total to be given.

Question		Distribution of points
1	*3 points*	1 point for the correct calculation of velocity in the x direction in km/sec 1 point for the correct calculation of velocity in the y direction in km/sec 1 point for the correct conversion to m/sec
2	*2 points*	1 point for the correct magnitude 1 point for the correct direction
3	*1 point*	1 point for the correct calculation of angular velocity
4	*2 points*	1 point for the correct calculation of acceleration in the x direction 1 point for the correct calculation of acceleration in the y direction
5	*2 points*	1 point for the correct magnitude 1 point for the correct direction
6 a	*2 points*	1 point for the conversion to meters 1 point for the correct calculation of force
6 b	*1 point*	1 point for the correct calculation of velocity
7	*1 point*	1 point for discussing the acceleration in problem 3 (only considering the **x** and **y** velocity and the **z** component are missing)
8	*1 point*	1 point for a discussion comparing the differences between linear acceleration and angular acceleration

APPENDIX A

SOLUTION KEY

STATE VECTORS

How are Position, Velocity and Acceleration Determined for the International Space Station (ISS)?

The Motion Control System is partly monitored and controlled by the Trajectory Operations Officer (TOPO) flight controller. The TOPO flight controller works in the Mission Control Center (MCC) for the International Space Station (ISS), along with a team of other flight controllers. These flight controllers are responsible for planning and tracking the current location and destination of the ISS. To learn more, explore and interact with the 3D ISS Mission Control Center on the ISS*Live!* website at http://spacestationlive.jsc.nasa.gov.

The position of the ISS is determined by a coordinate system that maps location in **x**, **y** and **z** state vectors. The change in these vectors allows us to follow the position, velocity and acceleration of the ISS and predict its location over time. To understand how to calculate the state vector, it is helpful to know how it was calculated historically.

Throughout time, astronomers have used a three–dimensional Cartesian coordinate system to identify positions in space. The origin of this coordinate system is located at the center of the Earth. The z–axis is defined as the line that runs through the North Pole. The x–axis and y–axis both lie on the plane formed by Earth's equator, with the x–axis pointing toward the vernal equinox.

Every year, there are two equinoxes—one in the spring (the vernal equinox) and one in the fall (the autumnal equinox). An equinox occurs when the sun passes directly over the equator of the Earth causing nearly equal amounts of daylight and night. The direction of the x–axis is always drifting because the Earth is always moving (rotating about the polar axis and orbiting the sun). For this reason, it is necessary to fix the orientation of the x–axis at a particular moment in time. The ISS uses the J2000 coordinate system that is based on the orientation of the Earth as it was midnight of January 1, 2000 (GMT).

Visit the TOPO console display in the 3D MCC environment and view the state vector data to answer the following questions.

1. Using ISS live data, determine the average velocity in meters per second (m/s) of the ISS in a 5–second time span in both the x and y directions.

Time (sec)	**x** position (km)	**y** position (km)
22:02:34	4,246.00	1,331.50
22:02:39	4,242.14	1,406.29

$$\Delta t = 22:02:39 - 22:02:34 = 5\,\text{sec}$$

$$\Delta S_x = 4{,}246.00 - 4{,}242.14 = 3.86\,km$$

$$\Delta S_y = 1{,}406.29 - 1{,}331.50 = 74.79\,km$$

$$\nu = \frac{\Delta S}{\Delta t}$$

$$\nu_x = \frac{3.86\,km}{5\,\text{sec}} \cdot \frac{1{,}000\,m}{1\,km} = 772\,\frac{m}{sec}$$

$$\nu_y = \frac{74.79\,km}{5\,\text{sec}} \cdot \frac{1{,}000\,m}{1\,km} = 14{,}960\,\frac{m}{sec}$$

2. Based on the average velocity of both the **x** and **y** positions, calculate both the magnitude and direction of the ISS velocity.

$$\nu = \sqrt{(\nu_x)^2 + (\nu_y)^2}$$

$$\nu = \sqrt{\left(772\frac{\text{m}}{\text{sec}}\right)^2 + \left(14{,}960\frac{\text{m}}{\text{sec}}\right)^2} = 15{,}000\,\frac{\text{m}}{\text{sec}}$$

$$\tan(\theta) = \frac{14{,}960\frac{\text{m}}{\text{sec}}}{772\frac{\text{m}}{\text{sec}}}$$

or

$$\left(14{,}960\frac{\text{m}}{\text{sec}}\right)^2 = \left(772\,\frac{\text{m}}{\text{sec}}\right)^2 + \left(15{,}000\,\frac{\text{m}}{\text{sec}}\right)^2 - (2)\left(772\frac{\text{m}}{\text{sec}}\right)\left(15{,}000\frac{\text{m}}{\text{sec}}\right) Cos(\theta)$$

$$\theta = 87^\circ$$

3. Assuming the Earth's radius is 6,371 km and the elevation of the ISS is 350 km, calculate the angular velocity of the ISS during the time interval.

$$\omega = \frac{\nu}{R}$$

$$\omega = \frac{15{,}000\,\frac{m}{sec}}{6{,}371\,km + 350\,km\,\left(\frac{1{,}000\,m}{1\,km}\right)} = 0.00223\,\frac{rad}{sec}$$

4. The ISS is continuously changing direction; therefore, it is accelerating in the x, y and z direction. Calculate the acceleration in both the x and y directions for a 5–second interval.

Time (sec)	dx/dt (m/sec)	dy/dt (m/sec)
02:26:22	4,201.02	4,696.04
02:26:27	3,507.77	5,985.19

$$\Delta t = 2:26:27 - 2:26:22 = 5\,\text{sec}$$

$$\Delta \nu_x = 3{,}507.77 - 4{,}201.02 = -693.25\,\frac{m}{sec}$$

$$\Delta \nu_y = 5{,}985.19 - 4{,}696.04 = 1{,}289.15\,\frac{m}{sec}$$

$$a = \frac{\Delta \nu}{\Delta t}$$

$$a_x = \frac{-693.25\,\frac{\text{m}}{\text{sec}}}{5\,\text{sec}} = -138.65\,\frac{\text{m}}{\text{sec}^2}$$

$$a_y = \frac{1{,}289.15\,\frac{m}{sec}}{5\,sec} = 257.83\,\frac{m}{sec^2}$$

5. Based on the above time interval, determine the acceleration.

$$a = \sqrt{(a_x)^2 + (a_y)^2}$$

$$a = \sqrt{\left(-193.25\,\frac{m}{\text{sec}^2}\right)^2 + \left(257.83\,\frac{m}{\text{sec}^2}\right)^2} = 322.21\,\frac{\text{m}}{\text{sec}^2}$$

$$\tan(\theta) = \frac{257.83\,\frac{\text{m}}{\text{sec}^2}}{-138.65\,\frac{\text{m}}{\text{sec}^2}}$$

or

$$\left(257.83\frac{\text{m}}{\text{sec}^2}\right)^2=\left(-138.65\frac{\text{m}}{\text{sec}^2}\right)^2+\left(322.21\frac{\text{m}}{\text{sec}^2}\right)^2-(2)\left(-138.65\frac{\text{m}}{\text{sec}^2}\right)\left(322.21\frac{\text{m}}{\text{sec}^2}\right)Cos(\theta)$$

$$\theta=129.28°$$

6. The mass of the ISS is approximately 450,000 kg, and as established earlier, the altitude is 350 km. Based on this information, calculate the following:

   a. The Earth has a mass of 5.974 x 10^{24} kg. Determine the relative force that exists between the ISS and the Earth?

$$F_G=\frac{\text{G}\,\text{m}_1\,\text{m}_2}{r^2}$$

$$F_G=6.67x10^{-11}\frac{\text{N}\cdot m}{\text{kg}^2}\cdot\frac{450{,}000\,\text{kg}\cdot5.974x10^{24}\,\text{kg}}{(6{,}721{,}000\,\text{m})^2}=3.970x10^6\,\text{N}$$

   b. In order maintain orbit at this altitude, calculate the minimum velocity that the ISS must travel?

$$F_{centripetal}=m\frac{\nu^2}{r}$$

$$3.970x10^6\,\text{N}=\frac{450{,}000\,kg\cdot\nu^2}{6{,}721{,}000\,\text{m}}$$

$$\nu^2=5.929x10^7\,\frac{m^2}{\text{sec}^2}$$

$$\nu=7{,}700.\frac{m}{sec}$$

7. The minimum velocity (calculated in question 6.b.) is different from that which was calculated earlier (in question 2). Explain what may account for the difference.

   The velocity calculations in question 6.b. take into account the **x**, **y** and **z** positions, while the velocity calculation for question 2 only considers the **x** and **y** positions.

8. Determine the centripetal acceleration of the ISS.

$$a_c=\frac{\nu^2}{r}$$

$$a_c=\frac{\left(7{,}700\frac{m}{sec}\right)^2}{6{,}721{,}000\,m}=8.82\frac{m}{sec^2}$$

APPENDIX B

STUDENT WORKSHEET

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2. Based on the average velocity of both the **x** and **y** positions, calculate both the magnitude and direction of the ISS velocity.

3. Assuming the Earth's radius is 6,371 km and the elevation of the ISS is 350 km, calculate the angular velocity of the ISS during the time interval.

4. The ISS is continuously changing direction; therefore, it is accelerating in the x, y and z direction. Calculate the acceleration in the x and y directions for a 5–second interval.

5. Based on the above time interval, determine the acceleration.

6. The mass of the ISS is approximately 450,000 kg, and as established earlier, the altitude is 350 km. Based on this information, calculate the following:

    a. The Earth has a mass of 5.974×10^{24} kg. Determine the relative force that exists between the ISS and the Earth?

    b. In order maintain orbit at this altitude, calculate the minimum velocity that the ISS must travel?

7. The minimum velocity (calculated in question 6.b.) is different from that which was calculated earlier (in question 2). Explain what may account for the difference.

8. Determine the centripetal acceleration of the ISS.